# Exercise 14

For the following exercises, find a domain on which each function $f$ is one-to-one and non-decreasing. Write the domain in interval notation. Then find the inverse of $f$ restricted to that domain.

$$f(x) = (x-6)^2$$

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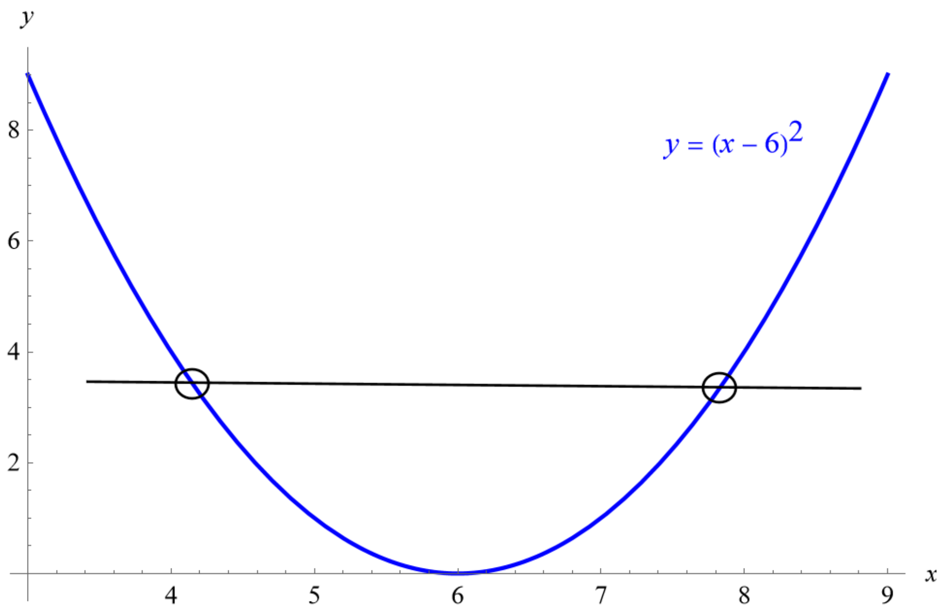
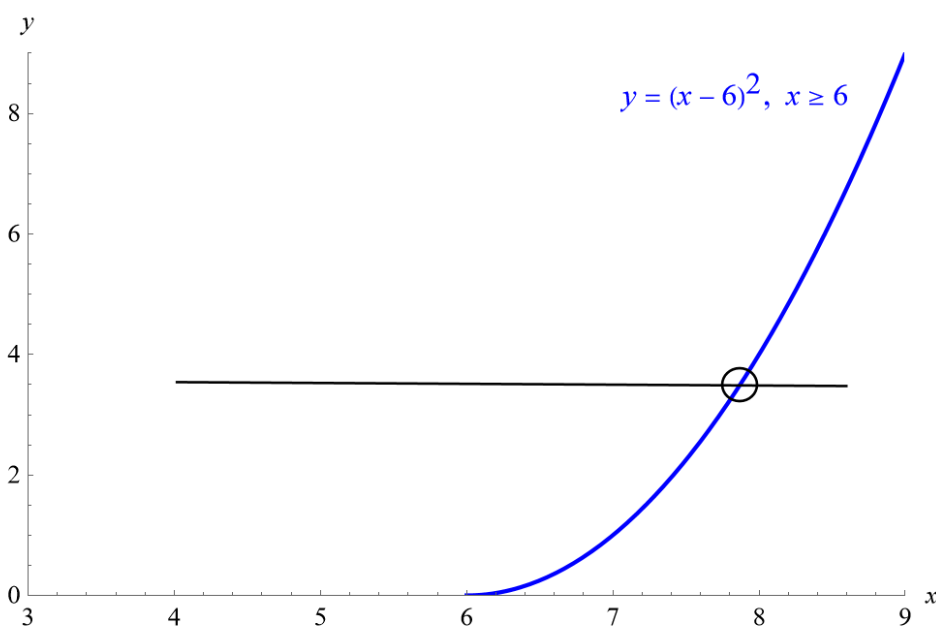
### Solution

This function is not one-to-one because it fails the horizontal line test.




But it can be made one-to-one by taking the restriction of $f(x)$ to $x \geq 6$.



The domain on which $f(x) = (x-6)^2$ is one-to-one and non-decreasing is $[6, \infty)$. To find the inverse, switch $x$ and $y$.

$$x = (y-6)^2$$

Solve for $y$. Take the square root of both sides.

$$\sqrt{x} = \sqrt{(y-6)^2}$$

Since there's an even power under an even root and the result is odd, an absolute value sign is needed.

$$\sqrt{x} = |y-6|$$

Remove the absolute value sign by placing $\pm$ on the left side.

$$\pm\sqrt{x} = y-6$$

Add 6 to both sides.

$$y = \pm\sqrt{x} + 6$$

In order to decide whether to choose the plus or minus sign, notice that $y$ originally came from $x$, which has the domain $[6, \infty)$. Choosing the minus sign would allow values of $y$ less than 6. Therefore, the inverse function is

$$f^{-1}(x) = \sqrt{x} + 6.$$